Nama : Mohammad Febryan Khamim

NRP : 5002221072

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Tugas 3 PDP (F)

2.1 Selesaikan PDP $u_x + u_y = 1$, dengan kondisi awal $u(x,0) = f(x)$

Cara 1 : Metode Lagrange

$u_x + u_y = 1$, $u(x,0) = f(x)$

$a = 1$; $b = 1$; $c = 1$

Diperoleh :

$$\frac{dx}{1} = \frac{dy}{1} = \frac{du}{1}$$

→ $dx = dy$

$\int dx = \int dy$

$x = y + C_1$

$x - y = C_1$

→ $dy = du$

$\int dy = \int du$

$y = u + C_2$

$C_2 = y - u$

Didapatkan PUPD-nya adalah :

PUPD : $F(x-y, y-u) = 0$

ekivalen dengan

$y - u = F(x-y)$

$u = y - F(x-y)$ ..... (1)

⇒ Substitusi kondisi awal :

$u(x,0) = f(x)$

$0 - F(x) = f(x)$

$-F(x) = f(x)$

$F(x-y) = -f(x-y)$

Substitusi ke (1)

$u = y + f(x-y)$

sehingga,

Persamaan Partikulir Persamaan Diferensial (PPPD) : $u = y + f(x-y)$

Cara 2 : Metode Karakteristik

$u_x + u_y = 1$, kondisi awal $u(x,0) = f(x)$

$u(x,0) = f(x)$

$x(0,s) = s$ $y(0,s) = 0$ $u(0,s) = f(s)$

$x(t,s) = \int 1 \, dt = t + f_1(s)$

$y(t,s) = \int 1 \, dt = t + f_2(s)$

$z(t,s) = \int 1 \, dt = t + f(s)$

→ $x(t,s) = t + f_1(s)$, $x(0,s) = s$

⇒ $s = 0 + f_1(s)$, sehingga $f_1(s) = s$

maka diperoleh : $x(t,s) = t + s$

→ $y(t,s) = t + f_2(s)$, $y(0,s) = 0$

⇒ $0 = 0 + f_2(s)$, sehingga $f_2(s) = 0$

maka diperoleh : $y(t,s) = t$

→ $u(t,s) = t + f_3(s)$, $u(0,s) = f(s)$

$f(s) = 0 + f_3(s)$, sehingga $f_3(s) = f(s)$

maka diperoleh : $u(t,s) = t + f(s)$.

※ $y = t$

$x = t + s$

$= y + s$

$s = x - y$

Maka : $u(x,y) = y + f(x-y)$

2.2 Selesaikan persamaan $xu_x + (x+y)u_y = 1$ dengan kondisi awal $u(1,y) = y$. Apakah solusi terdefinisi dimana-mana?

▶ Cara 1 : Metode Lagrange

$xu_x + (x+y)u_y = 1$, kondisi awal $u(1,y) = y$

diperoleh $a = x$; $b = (x+y)$; $c = 1$

$$\frac{dx}{x} = \frac{dy}{x+y} = \frac{dU}{1}$$

• $\dfrac{dx}{x} = \dfrac{dy}{x+y}$

$$\frac{(x+y)\,dx}{x} = dy$$

$$\left(1 + \frac{y}{x}\right) dx = dy$$

misal : $v = \dfrac{y}{x}$

$\rightarrow y = vx$

$dy = v\,dx + x\,dv$

$\rightarrow (1+v)\,dx = v\,dx + x\,dv$

$$(1+v-v)\,dx = x\,dv$$

$$dx = x\,dv$$

$$\int \frac{1}{x}\,dx = \int dv$$

$$\ln x = v + C_1$$

$$C_1 = \ln x - \frac{y}{x}$$

• $\dfrac{dx}{x} = \dfrac{dU}{1}$

$$\int \frac{1}{x}\,dx = \int du$$

$$\ln x = u + C_2$$

$$C_2 = \ln x - u$$

Diperoleh : PUPD : $F(C_1, C_2) = 0$

$$f\left(\ln x - \frac{y}{x}, \ln x - u\right) = 0$$

$$\ln x - u = f\left(\ln x - \frac{y}{x}\right)$$

$$u = \ln x - f\left(\ln x - \frac{y}{x}\right)$$

Solusi awal : $u(1,y) = y$, maka

$$u(1,y) = \ln 1 - f\left(0 - \frac{y}{1}\right)$$

$$= 0 - f(-y)$$

Sehingga, $f\left(\frac{y}{x} - \ln x\right) = \frac{y}{x} - \ln x$

jadi didapatkan penyelesaian khusus

$$u(x,y) = \ln x + f\left(\frac{y}{x} - \ln x\right)$$

$$u(x,y) = \ln x + \frac{y}{x} - \ln x$$

$$u(x,y) = y/x$$ //

▶ Cara 2 : Metode Karakteristik

$xU_x + (x+y)U_y = 1$

Dengan menggunakan parameter :

• $x(t,s) = x$ $\qquad u(t,s) = 1$

• $y(t,s) = x + y$

Karena kondisi awal $U(1,y) = y$, maka

$x(0,s) = 1$ $\qquad U(0,s) = s$

$y(0,s) = s$

$\Rightarrow U_t(t,s) = 1$

$$U(t,s) = \int dt = t + f_1(s)$$

$$U(0,s) = s$$

$$= 0 + f_1(s)$$

Jadi, $f_1(s) = s$

$$U(t,s) = t + s \quad \ldots (1)$$

Karena $x(0,s) = 1$ maka

$$y_t = x + y$$

$$y_t = e^t + y$$

$$y_t - y = e^t$$

$P(t) = -1$ $\qquad Q(t) = e^t$

$$v = e^{\int P(t)\,dt}$$

$$= e^{\int -1\,dt}$$

$$= e^{-t}$$

Penyelesaian PDB :

$yv = \int v\, Q(t)\, dt$

$ye^{-t} = \int e^{-t} e^{t}\, dt$

$ye^{-t} = \int dt$

$ye^{-t} = t + f_2(s)$

$y(t,s) = e^{t}\left(t + f_2(s)\right)$

$y(0,s) = s \rightarrow e^{0}(0 + f_2(s)) = s$

$f_2(s) = s$

maka $y(t,s) = e^{t}(t+s)$ .... (2)

Dari (1) dan (2) diperoleh

$y = e^{t}(t+s)$

$= x \cdot u$

$u = \frac{y}{x}$

PKPD : $u(x,y) = \frac{y}{x}$

Ket. Solusi khusus tersebut terdefinisi dimana mana selain $x \neq 0$